COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-62/46

A. Fujii: KAON PRODUCTION BY HIGH ENERGY NEUTRINOS

Nota interna: n° 141
6 Giugno 1962

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We have calculated the cross section of the charged kaon production induced by high energy neutrinos,

$$\bar{\nu}_\ell + p \rightarrow p + \ell^+ + K^-,$$

in the peripherical approximation[1]. The lepton associated with the reaction is either an electron or a muon. The threshold neutrino energy in the laboratory system is 0.62 GeV and 0.79 GeV for the electron and muon mode respectively.

In this reaction most of the incident energy is carried away by the lepton, leaving the strongly interacting particles in relatively low energy state. Then the single pion exchange mechanism shown in fig. 1 will dominate, and the upper "vertex" can be related to the form factors of the K^+_{l3} decay

$$K^+ \rightarrow \pi^0 + \ell^+ + \nu_\ell .$$

The other pole diagram shown in fig. 2 is simply neglected, because the appearance of the heavy virtual particle is improbable in low energy region.

The cross section formula for a diagram of the type in fig. 1 has been derived by several authors[2)3)4)].

The partial cross section of the above process reads

$$\frac{d^2\sigma}{d\omega^2 d\Delta^2} = \frac{1}{(2\pi)^2}\frac{g^2}{4\pi}\frac{1}{16(W^2-m_p^2)^2}\frac{\Delta^2(\Delta^2+\omega^2)}{\omega^2(\Delta^2+m_\pi^2)^2}\sqrt{\omega^4-2(m_K^2+m_\ell^2)\omega^2+(m_K^2-m_\ell^2)^2}$$

(1) $$\times\Big[|f|^2\Big(\omega^2-m_K^2-m_\ell^2\,\frac{2\omega^2+m_K^2-m_\ell^2}{\omega^2}\Big)$$

$$+2Re(fg^*)m_\ell^2\,\frac{\omega^2+m_K^2-m_\ell^2}{\omega^2}+|g|^2 m_\ell^2\,\frac{\omega^2-m_K^2+m_\ell^2}{\omega^2}\Big]$$

where

$$\omega^2=-(q_1+q)^2\,,\quad \Delta^2=(p_2-q_2)^2,$$

W is the total energy in the over-all center of mass system and is related to the neutrino energy of the laboratory system E_ν by

$$E_\nu=\frac{W^2-m_p^2}{2m_p}\,,$$

$\frac{g^2}{4\pi}$ is the renormalized π^0-proton coupling constant. f and g are the form factors of the K^+_{13} decay defined by

$$\langle\pi^0|j_\mu|K^+\rangle=\frac{1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_K}}\frac{1}{\sqrt{2\omega_\pi}}\Big[\frac{f}{2}(p_K+p_\pi)_\mu+\Big(\frac{f}{2}-g\Big)(p_K-p_\pi)_\mu\Big],$$

where p and ω stand for the four-momentum and energy of the specified meson, and are in general functions of $(p_K-p_\pi)^2$. Assuming that they are slowly varying functions and approximating them by constants we obtain the decay rate of K^+_{13} as [5])

$$w(K^+_{\ell 3})=\frac{1}{(2\pi)^3}\frac{m_K^5}{16}\Big[|f|^2\big(-\alpha\beta I_{-1}+(1+2\alpha\beta)I_0-(2+\alpha\beta)I_1+I_2\big)$$

(2) $$-4Re(fg^*)\Big(\frac{m_\ell}{m_K}\Big)^2(I_{-1}-2I_0+I_1)$$

$$+|g|^2\Big(\frac{m_\ell}{m_K}\Big)^2\big(-\alpha\beta I_{-2}+(1+2\alpha\beta)I_{-1}-(2+\alpha\beta)I_0+I_1\big)\Big]$$

where

$$\alpha = \frac{m_\pi + m_l}{m_K} , \qquad \beta = \frac{m_\pi - m_l}{m_K} ,$$

and

$$I_n = \int_{\alpha^2}^{1} x^n \sqrt{(x-\alpha^2)(x-\beta^2)}\, dx .$$

The integral I_n's are explicitly computed and tabulated in Reference (5). In particular if the lepton mass is put equal to zero, i.e. $\beta = \alpha$,

$$(3) \qquad w(K^+_{e3}) = \frac{1}{(2\pi)^3}\frac{m_K^5}{8}|f|^2\left(\frac{1}{24} - \frac{1}{3}\alpha^3 + \frac{1}{3}\alpha^6 - \frac{1}{24}\alpha^8 - \alpha \ln\alpha\right).$$

We obtain the total cross section first by integrating (1) with respect to Δ^2 from Δ^2_{min} to Δ^2_{max},

$$(4) \qquad \Delta^2_{\substack{max\\min}} = \frac{1}{2W^2}\left[(W^2-m_p^2)^2 - (W^2+m_p^2)\omega^2 \pm (W^2-m_p^2)\sqrt{\omega^4 - 2(W^2+m_p^2)\omega^2 + (W^2-m_p^2)^2}\right],$$

then integrating the resultant with respect to ω^2 from $(m_K + m_l)^2$ to $(W - m_p)^2$.

It is more convenient to transcribe (1), (2), (3) and (4) with the dimensionless variables defined by

$$x = \frac{\omega^2}{m_p^2} , \qquad y = \frac{\Delta^2}{m_p^2} , \qquad \xi = \frac{W^2}{m_p^2}$$

$$\lambda = \left(\frac{m_l}{m_p}\right)^2 , \quad \mu = \left(\frac{m_\pi}{m_p}\right)^2 , \quad K = \left(\frac{m_K}{m_p}\right)^2$$

We further assume that the time reversal invariance holds for $\langle \pi^0 | j_\mu | K^+ \rangle$ and that the form factors are approximated by constants, so that they are replaced by two dimensionless real parameters Γ^2 and η,

$$\Gamma^2 = |f|^2 m_p^4 , \qquad \eta = \frac{g}{f} .$$

Then

$$\frac{d^2\sigma}{dx\,dy} = \sigma_0 \frac{1}{(\xi-1)^3}\int_{(\sqrt{K}+\sqrt{\lambda})^2}^{(\sqrt{\xi}-1)^2} dx \int_{y_{min}(x)}^{y_{max}(x)} dy\, \frac{y(x+y)}{x(y+\mu)^2}\sqrt{x^2-2(K+\lambda)x+(K-\lambda)^2}$$

$$\times\left[x-K-\lambda\left(\frac{2x+K-\lambda}{x} - 2\eta\frac{x+K-\lambda}{x} - \eta^2\frac{x-K+\lambda}{x}\right)\right]$$

$$\sigma_0 = \frac{1}{(2\pi)^2}\,\frac{g^2}{4\pi}\,\frac{f^2}{16}\,\frac{1}{m_\rho^2}$$

$$y_{\substack{max\\min}}(x) = \frac{1}{2\xi}\left[(\xi-1)^2-(\xi+1)x \pm (\xi-1)\sqrt{x^2-2(\xi+1)x+(\xi-1)^2}\right]$$

$$w(K_{e3}^+) = \frac{1}{(2\pi)^3}\,\frac{f^2K^2}{16}\,m_K\left[\left(-\alpha\beta I_{-1}+(1+2\alpha\beta)I_0-(2+\alpha\beta)I_1+I_2\right)\right.$$

$$\left. -4\eta\frac{\lambda}{K}\left(I_{-1}-2I_0+I_1\right)+\eta^2\frac{\lambda}{K}\left(-\alpha\beta I_{-2}+(1+2\alpha\beta)I_{-1}-(2+\alpha\beta)I_0+I_1\right)\right]$$

$$w(K_{e3}^+) = \frac{1}{(2\pi)^3}\,\frac{f^2K^2}{8}\,m_K\left[\frac{1}{24}-\frac{1}{3}\alpha^2+\frac{1}{3}\alpha^6-\frac{1}{24}\alpha^8-\alpha^4\ln\alpha\right].$$

The y-integration can be performed analytically, but the final x-integration must be done numerically, preferably by a machine.

To estimate the order of magnitude of the cross section quickly, we put $\lambda = 0$ and assume $\xi \gg 1$ in the x-integrand. This choice of parameters corresponds to the high energy limit where the lepton mass is practically negligible. Eqn. (5) becomes then

$$(10)\qquad \sigma(\xi) = \sigma_0\frac{1}{(\xi-1)^2}\int_K^{(\sqrt{\xi}-1)^2} dx \int_0^{\xi-x} dy\, \frac{y(x+y)(x-K)^2}{x(y+\mu)^2},$$

and yields a lengthy formula as an explicit function of ξ, K and μ after a series of elementary integrations. Equating (9) with the experimental decay rate of K_{e3}^+, 3.43×10^6 sec^{-1}, we find

$$\Gamma^2 = 4.97 \times 10^{-12} \quad ,$$

therefore

$$\sigma_0 = 5.20 \times 10^{-41} \text{ cm}^2$$

with $g^2/4\pi = 15$. As an example if we take $E_\nu = 10$ GeV,

$$\xi = 22.32 \;, \qquad k = 0.2771 \;, \qquad \mu = 0.0207 \;,$$

and we obtain

$$\sigma = \sigma_0 \times 1.22 \times 10 = 6.3 \times 10^{-40} \text{cm}^2 \;,$$

On the other hand at $E_\nu = 10$ GeV, the cross section of the "elastic" process

$$\bar{\nu}_e + p \rightarrow n + e^+$$

is about 68×10^{-40} cm^2 6), and that of the pion production process

$$\bar{\nu}_e + p \rightarrow n + e^+ + \pi^-$$

is about 40×10^{-40} cm^2 7).

The machine integration of Eqn. (5) for several choices of the parameters ξ and η, and the machine integration with dispersion theoretic form factors are in progress.

The author would like to express his thanks to Professors R. Gatto, N. Cabibbo and B. Vitale for discussions and comments.

References

1) F. Salzman and G. Salzman, Phys. Rev. Letters 5, 377 (1960); Phys. Rev. 125, 1703 (1962)
2) D. Boccaletti and F. Selleri, Nuovo Cimento 22, 1099 (1961)
3) P.G.O. Freund and A. Fujii, Nuovo Cimento 23, 848 (1962)
4) E. Ferrari and F. Selleri, "Pheripheral Model for Inelastic Processes", CERN Preprint, March 1962. This paper includes the complete bibliography of the subject.
5) A. Fujii and M. Kawaguchi, Phys. Rev. 113, 1156 (1959)
6) Y. Yamaguchi, Prog. Theoret. Phys. 23, 1117 (1960)
7) N. Cabibbo and G. Da Prato, Nuovo Cimento, in press, and private communication.

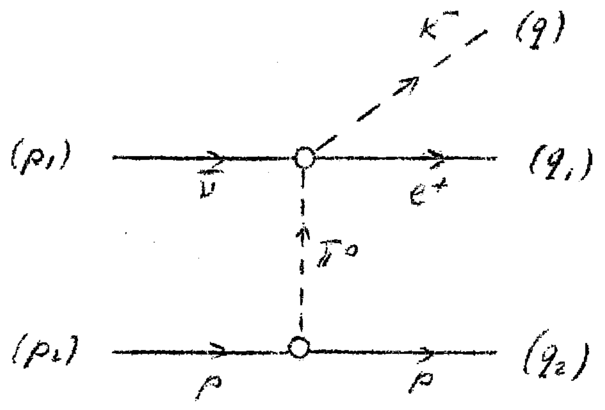

FIG. 1

Single pion exchange diagram. The letter inside the bracket stands for a four-momentum of the particle.

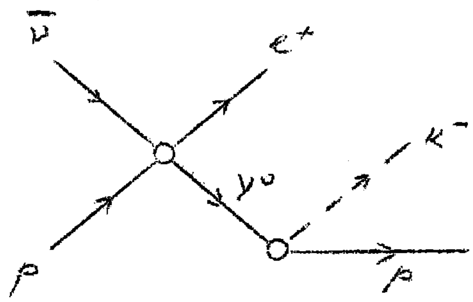

FIG. 2

Hyperon pole diagram.